Frequency Dependent Circuits: Active Electronic Filter Classification

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Abstract

This document contains information about experiments conducted for experiential learning of the effects electronic circuits composed of *passive* and *active* elements have on an input AC energy source. Various types of active filters formed by the combination of passive elements (i.e. resistors, capacitors, inductors) and the active element, a 741 op amp, are used in these experiments to eliminate input sinusoidal signals oscillating at a range of frequencies. To determine what frequencies are filtered by each permutation of elements in a circuit, a frequency sweep from 100Hz to 20kHz was applied to the AC source on four computer simulated RC and RL active circuits as well as on a physical implementation which produced similar results, as expected. The results of the frequency sweep reveal that each permutation of the RC and RL circuit elements will filter-out a different range of high frequencies effectively making them *lowpass filters*. Another permutation of these circuits filter-out a range of low frequencies effectively making them *highpass filters*. Some of these circuits are also able to filter-out frequencies above 1MHz and below 20kHz effectively making them *bandpass filters* when the half-power point is approximately -3dB. Before each circuit's *passband* frequencies, or the frequencies a circuit's output energy is at or above the half-power point which is no less than -3dB the input energy, are revealed, the effects of the input AC frequency on the output voltage and the phase shift of the output voltage relative to the input voltage are studied thoroughly for one of the RC circuits. This analysis reveals that the input frequency affects the phase shift and peak voltage of the output voltage.

I. Introduction

Filter circuits have the ability to limit the effect an input signal has on a circuit when the input signal frequency is not suitable for operating some device connected to the circuit. A practical application of filter circuits is found in commercial radios. There are only certain bands that audio signals are transmitted and received; thus, civilian radios are equipped with filters that output energy at these particular frequencies. To understand how circuits can be built to filter different frequencies of alternating current, this document covers four simple circuits composed of resistors, op amps, and capacitors or inductors. The gain introduced by these circuits will be displayed on bode plots which reveals the range of frequencies a circuit is capable of filtering.

II. Time-domain Analysis of an RC Circuit's Frequency Response

To understand how a circuit responds to various AC frequencies, an oscilloscope is used to measure the peak input voltage V_{in} at some point in time and the first occurrence of the peak output voltage V_{out} following V_{in}. For the RC Circuit in Figure 1, the oscilloscope reveals that V_{out}, which is the voltage across the capacitor, is greater when the input AC voltage oscillates at lower frequencies than at higher frequencies. This frequency response is an immediate indication that the circuit acts is potentially either a lowpass or *bandreject* filter; but the circuit's response to higher frequencies needs to be examined before rejecting the possibility of it being a bandreject filter. Observing the frequency response of the circuit through 20kHz or greater will reveal that the filter is in fact a lowpass filter because the current passes through almost unimpeded in only one *passband*, or in a small range of lower frequencies. From this observation, it can be deduced that the capacitor will act like a short circuit at infinite frequency.

The oscilloscope also reveals that there is a phase shift between the output voltage and the input voltage. In this circuit, the voltage across the capacitor peaks after the input voltage because the capacitor needs to charge up. This delay explains why the voltage across a capacitor cannot change immediately. This effect caused by the capacitor can also be observed through the peak voltage readings at different frequencies because when the input frequency is too high, the capacitor may not have enough time to reach its maximum voltage before receiving negative charge from the AC source. That is why there is no phase shift between the input and output voltage when the AC frequency is at, for example, 1Hz because the capacitor is able to charge in the same amount of time it takes for the AC source voltage to peak.

$$Frequency = \frac{\omega}{2\pi} \tag{1}$$

$$Phase\ shift\ =\ (360^{\circ})(Td)(Frequency) \tag{2}$$

$$Td\ =\ Time_{V_{in}} - Time_{V_{out}} \tag{3}$$

Fig. 1: RC Circuit #1

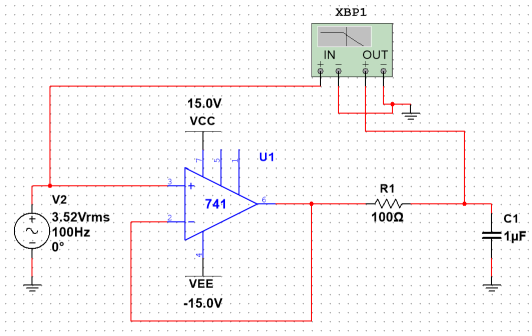


TABLE I: RC circuit frequency sweep analysis

RC Circuit Analysis							
R_a	C_a	ω	Frequency	V_{in} peak	V_{out} peak	Td	Phase Shift
470Ω	1μF	212.77 rad/s	33.86Hz	4.97V	4.95V	0.911ms	-11.10°
470Ω	1μF	1063.83 rad/s	169.31Hz	4.98V	4.43V	0.511ms	-31.15°
470Ω	1μF	2127.66 rad/s	338.63Hz	4.98V	3.51V	0.372ms	-45.35°
470Ω	1μF	4255.32 rad/s	677.26Hz	4.97V	2.21V	0.241ms	-58.76°
470Ω	1μF	21276.6 rad/s	3386.28Hz	4.97V	0.267V	0.073ms	-88.99°

III. Frequency-domain Analysis of RC and RL Active Circuit Filters

To determine the passband frequencies a circuit is ideally suited for, a *cutoff frequency* also known as the *corner frequency* is defined as $\omega_c = M_0/\sqrt{2}$ where M_0 is the magnitude, or maximum amount of power that is emitted by the circuit. A frequency-domain analysis using a generator that quickly "sweeps" a range of AC frequencies at 1 Volt reveals the ideal corner frequency and range of the passband(s), or *bandwidth*, capable of being unimpeded by the filter circuits. For the circuits below, this analysis is performed by sweeping the AC source from 100Hz to 20kHz. The results are graphed on bode plots where the magnitude (in decibels) and phase (in degrees) of a circuit's frequency response are plotted with respect to a semi-logarithmic scale of frequencies.

A. Analysis of voltage across the capacitor in active RC Circuit #1

The capacitor causes the output voltage to lag the input voltage thus the phase plot range is from 0° to -90°. The oscilloscope readings and calculations listed in Table 1 do not agree with all the values in the graphs, but the AC sweep analysis supports the original conclusion that this is a lowpass filter circuit.
The voltage gain introduced by this circuit can be summarized by the following transfer function:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} \tag{4}$$

Fig. 2: Frequency Response of RC Circuit #1

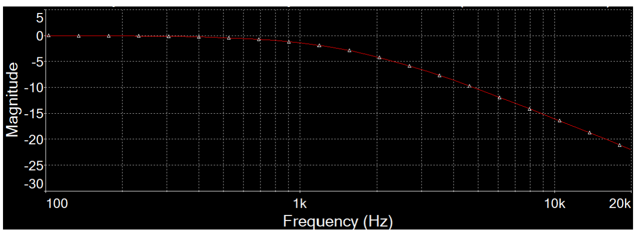
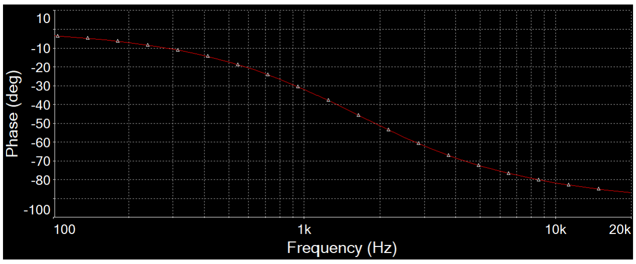


Fig. 3: Phase Response of RC Circuit #1



B. Analysis of voltage across the resistor in active RC Circuit #2

This circuit is similar to RC circuit #1 except that the positions of the capacitor and resistor have been swapped. Because the output voltage is taken off from across the resistor, the output voltage does not lag the input voltage. The readings in the bode plots characterize the circuit as a highpass filter.
The voltage gain introduced by this circuit can be summarized by the following transfer function:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC} \tag{5}$$

Fig. 4: RC Circuit #2

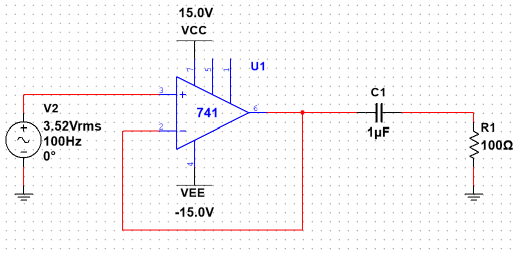


Fig. 5: Frequency Response of RC Circuit #2

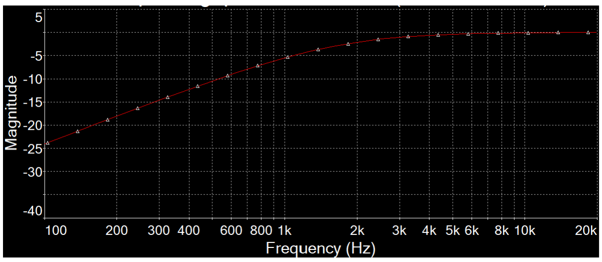
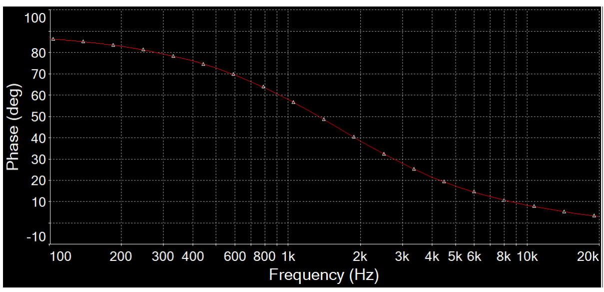


Fig. 6: Phase Response of RC Circuit #2



C. Analysis of voltage across the resistor in active RL Circuit #1

The voltage across the inductor increases rapidly when a current is applied to it, but the current does not immediately flow through it. This causes there to be low voltage across the subsequent resistor when the input AC frequency is too high because there is not enough time for a sufficient amount of current to flow through the inductor. As a result, the inductor acts like an open circuit at high AC frequencies. The readings in the bode plots characterize the circuit as a lowpass filter.
The voltage gain introduced by this circuit can be summarized by the following transfer function:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{j\omega L}{R + j\omega L} \tag{6}$$

Fig. 7: RL Circuit #1

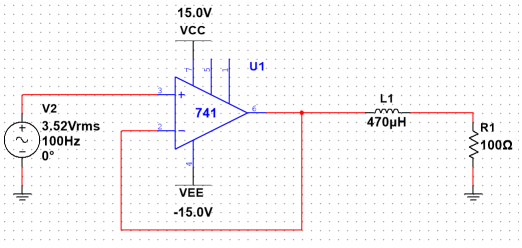


Fig. 8: Frequency Response of RL Circuit #1

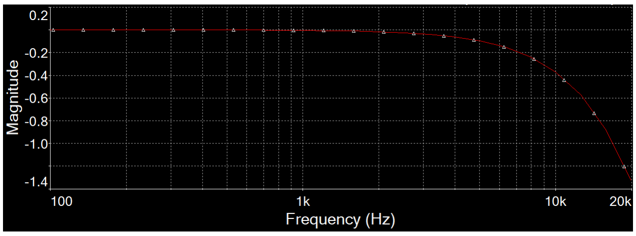
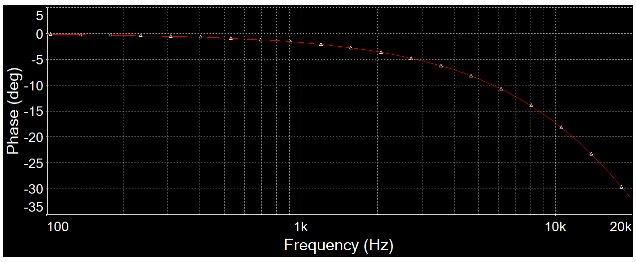


Fig. 9: Phase Response of RL Circuit #1



D. Analysis of voltage across the inductor in active RL Circuit #2

When the voltage is taken from the inductor in the RL circuit, the voltage across the inductor increases as the input AC frequency is increased. The inductor will act like an open circuit when the input frequency is very high; however, it will begin to diminish when the input frequency surpasses 10MHz. Allegedly, op amps are not suitable for filtering high frequencies thus the circuit may be characterized as a bandpass filter if high frequencies are included in the AC sweep analysis. In the analysis performed the readings in the bode plots characterize the circuit as a highpass filter.
The voltage gain introduced by this circuit can be summarized by the following transfer function:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L} \tag{7}$$

Fig. 10: RL Circuit #2

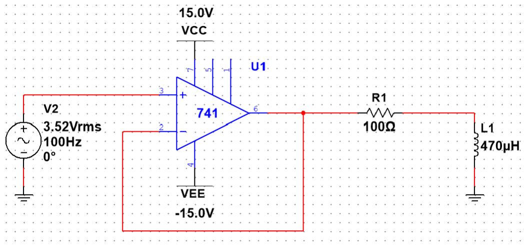


Fig. 11: Frequency Response of RL Circuit #2

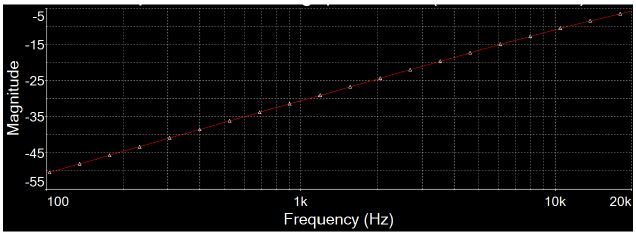
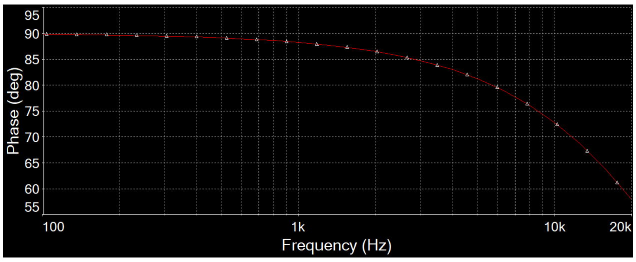


Fig. 12: Phase Response of RL Circuit #2



IV. Conclusion

There are four types of filter circuits: lowpass, highpass, bandpass, and bandstop. A lowpass filter can be created when the output is taken off a capacitor in an RC circuit or off a resistor in an RL circuit. A highpass filter can be created when the output is taken off a resistor in an RC circuit or off an inductor in an RL circuit. A bandpass and bandstop filter can be created with an RLC circuit or an RC or RL circuit combined with an op amp. Although the circuits covered in this document contained op amps, the voltage was taken out of components that followed the op amp, thus the op amp had minimal effect. One of the active RL circuits however acted as a bandpass filter because it was able to attenuate very high frequencies; however, op amps are considered to be unreliable above 1MHz, therefore the circuit may considered as a highpass filter below 1MHz input signals. Learning about what combination of passive and active elements will produce which type of filter is important for designing a circuit that attenuates unwanted frequencies.